

stressed that any application of a two-dimensional water quality model be verified either through site-specific salinity or dye tracer data. Naturally, when performing field tracer experiments the time and length scales of the field phenomenon should be compatible with the time and length scales to be represented in the model. For example, a dye study lasting only a few hours is not valid for verification of a model using a daily computational time step. Similarly, a dye study confined to a small portion of a large lake or estuary will not allow for verification of the model over the entire system.

#### 2.3.4 Longitudinal Dispersive Transport in Estuaries

As previously discussed, longitudinal dispersion is the "effective diffusion" that occurs in one-dimensional mass transport equations that have been integrated over the cross sectional area perpendicular to flow. This one-dimensional approach to modeling has often been applied to tidal and nontidal rivers, and to estuaries.

The magnitude of the one-dimensional dispersion coefficient in estuaries and tidal rivers is determined in part by the time scale over which the simulation is performed. The time scale specifies the interval over which quantities that generally change instantaneously, such as tidal current, are averaged. For shorter time scales the simulated hydrodynamics and therefore water quality relationships are resolved in greater detail and hence, in such models, smaller dispersion coefficients are needed than in those which, for example, average hydrodynamics over a tidal cycle.

The magnitude of the dispersion coefficient can also be expected to change as a function of location within an estuary. Since the one-dimensional dispersion coefficient is the result of spatial averaging over a cross section perpendicular to flow, the greater the deviation between actual velocity and the area-averaged velocity, and between actual constituent concentrations and area-averaged concentrations, the larger will be the dispersion coefficient. These deviations are usually largest near the mouths of estuaries due to density gradients set up by the

interface between fresh and saline water. Strong tidal currents may also result in large dispersion coefficients.

Because of the time scale and location dependency of the dispersion coefficient, it is convenient to divide the discussion of dispersion into time varying and tidally averaged time expressions, and then to subdivide these according to estuarine location, i.e., the salinity intrusion region and the freshwater tidal region. The salinity intrusion region is that portion of the estuary where a longitudinal salinity gradient exists. The location of the line of demarcation between the salinity intrusion region and the freshwater tidal region varies throughout the tidal cycle, and also depends on the volume of freshwater discharge. It should also be noted that the freshwater tidal region can contain saline water, if the water is of uniform density throughout the region (TRACOR, 1971). There is at present no analytical method for predicting dispersion in the salinity intrusion region of estuaries. However, because of the presence of a conservative constituent (salinity), empirical measurements are easily performed. In the freshwater tidal region, analytical expressions have been developed, while empirical measurements become more difficult due to the lack of a naturally occurring conservative tracer. Empirical measurements can alternatively be based, however, on dye release experiments.

#### 2.3.4.1 Time Varying Longitudinal Dispersion

A model which is not averaged over the tidal cycle is more capable of representing the mixing phenomena since it represents the time varying advection in greater detail. However, the averaging effects of spatial velocity gradients (shear) and density gradients must still be accounted for. The specification of longitudinal dispersion coefficients is closely associated with the type of mathematical techniques used in a given model. Most of the model developments for one-dimensional representation of estuaries has occurred in the early 1970's, and the most prominent techniques are summarized below.

The "link-node" or network model developed originally by WRE (1972) and commonly known as the Dynamic Estuary Model (DEM) used the basic work of Feigner and Harris (1970) to describe the numerical dispersion in the constant density region of an estuary:

$$D_L = C_1 E^{1/3} L_e^{4/3} \quad (2-41)$$

where  $D_L$  = longitudinal dispersion coefficient, length<sup>2</sup>/time

$E$  = rate of energy dissipation per unit mass

$L_e$  = mean size of eddies participating in the mixing process

$C_1$  = function of relative channel roughness

For computational purposes, Feigner used the following simplification:

$$D_L = 0.042 |u| R \quad (2-42)$$

where  $R$  = hydraulic radius, ft

$|u|$  = absolute value of velocity, ft/sec

There exists no corresponding formulation for the longitudinal dispersion coefficient in the salinity intrusion regions of estuaries. Rather, a careful calibration procedure is required using available salinity data to prescribe the appropriate dispersion coefficients. Obviously, this approach somewhat restricts the predictive nature of such models since a substantial amount of empirical data is necessary for proper model application.

Similar versions of the DEM exist in one form or another. Not all versions, however, include the option for specification of longitudinal dispersion. This stems from the fact that considerable numerical dispersion occurs in the DEM from the first order, explicit, finite difference treatment of the advective transport terms. Feigner and Harris (1970) gave some comparisons of different weightings of the first order differencing in terms of trade-offs between numerical mixing, accuracy, and stability. Work on this problem has been done by Bella and Grenney (1970) and a numerical

estimate of this dispersion can be given by the following equation:

$$D_{\text{num}} = \frac{V}{2} \left[ (1-2\nu) \Delta x - V\Delta t \right] \quad (2-43)$$

where  $\nu$  represents the weighting coefficient assigned to the concentrations of two adjacent nodes.

This equation shows that the numerical dispersion is a function of  $\Delta x$ ,  $\Delta t$ , and the velocity,  $V$ , which is a function of location and time. This equation is useful for estimating the magnitude of numerical dispersion. It illustrates the lack of control that the modeler has over this phenomena in the DEM.

Daily and Harleman (1972) developed a network water quality model for estuaries which uses a finite element numerical technique. The hydraulics are coupled to the salinity through the density-gradient terms in the manner formulated by Thatcher and Harleman (1972). The high accuracy finite element Galerkin weighted residuals technique is relatively free of artificial numerical dispersion. The longitudinal dispersion formulation combines both the vertical shear effect and the vertical density-induced circulation effect through the following expression:

$$D(x,t) = K \left| \frac{\partial s}{\partial x} \right| + m D_T \quad (2-44)$$

where  $D(x,t)$  = temporally and spatially varying dispersion coefficient,  $\text{ft}^2/\text{sec}$ .

$g$  =  $s/s_0$  where  $s(x,t)$  is the spatial and temporal distribution of salinity, ppm

$s_0$  = ocean salinity, ppm

$g$  =  $x/L$

$L$  = length of estuary, ft (to head of tide)

$D_T$  = Taylor's dispersion coefficient in  $\text{ft}^2/\text{sec} = 77 u n R_h^{5/6}$

$u$  =  $u(x,t)$  tidal velocity,  $\text{ft}/\text{sec}$

- n = Manning's friction coefficient.
- $R_h$  = hydraulic radius, ft.
- K = estuary dispersion parameter in  $\text{ft}^2/\text{sec} = u_o L/1000$
- $u_o$  = maximum ocean velocity at the ocean entrance, ft/sec
- m = a multiplying factor for bends and channel irregularities

One-dimensional, time varying modeling using this expression has been performed for several estuaries, a recent example being an application (Thatcher and Harleman, 1978) to the Delaware Estuary wherein the time-varying calculations were made for a period of an entire year in order to provide a model for testing different water management policies.

For real time simulations in the constant density region of estuaries and tidal rivers, the following expression has been proposed (TRACOR, 1971):

$$D_L = 100 n U_{\max} R_H^{5/6} \quad (2-45)$$

- where  $D_L$  = longitudinal dispersion coefficient in the constant density region,  $\text{ft}^2/\text{sec}$
- n = Manning's roughness coefficient,  $\text{ft}^{1/6}$
- $U_{\max}$  = maximum tidal velocity, ft/sec
- $R_H$  = hydraulic radius, ft.

The determination of real time dispersion coefficients in the salinity intrusion region requires field data on salinity distribution. Once the field data have been collected, the magnitudes of the dispersion coefficients can be found by fitting the solution of the salinity mass transport equation to the observed data. As reported in TRACOR (1971), this technique has been applied to the Rotterdam Waterway, an estuary of almost uniform depth and width. The longitudinal dispersion coefficient was found to be a function of x, the distance measured from the mouth (ft), as follows:

$$D_L = 13000 \left(1 - \frac{x}{L}\right)^3 \quad (2-46)$$

where  $D_L$  = real time longitudinal dispersion coefficient in salinity intrusion region,  $\text{ft}^2/\text{sec}$

$L$  = length of entire tidal region of the estuary.

At the estuary mouth,  $D_L$  was found to be  $13,000 \text{ ft}^2/\text{sec}$  or  $40 \text{ mi}^2/\text{day}$  ( $1.2 \times 10^7 \text{ cm}^2/\text{sec}$ ) by using the technique described above. Under the same conditions in a constant density region, Equation (2-38) predicts  $D_L = 175 \text{ ft}^2/\text{sec}$ , or  $0.5 \text{ mi}^2/\text{day}$  ( $1.6 \times 10^5 \text{ cm}^2/\text{sec}$ ). This illustrates the large difference that can be expected between the real time dispersion coefficient in the salinity intrusion region of an estuary and in the constant density region. For more detailed discussions of real time longitudinal dispersion in estuaries, see Holley et al. (1970) and Fischer et al. (1979).

#### 2.3.4.2 Steady State Longitudinal Dispersion

For tidally averaged or net nontidal flow simulations, the dispersion coefficients must somehow include the effects of oscillatory tidal mixing which has been averaged out of the hydrodynamics representation. No known general analytical expressions exist for this coefficient. Hence, it is cautioned and emphasized that steady-state dispersion coefficients must be determined based on observed data, or based on empirical equations having parameters that are determined from observed data. This limitation exists for both the constant density and salinity intrusion regions of the estuary.

In their one-dimensional tidally averaged estuary model, Johanson et al. (1977) used an empirical expression, comprised of three principal components (tidal mixing, salinity gradient, and net freshwater advective flow) for the dispersion coefficient. The relative location in an estuary where each of these factors is significant, and their relative magnitudes, are shown in Figure 2-7.

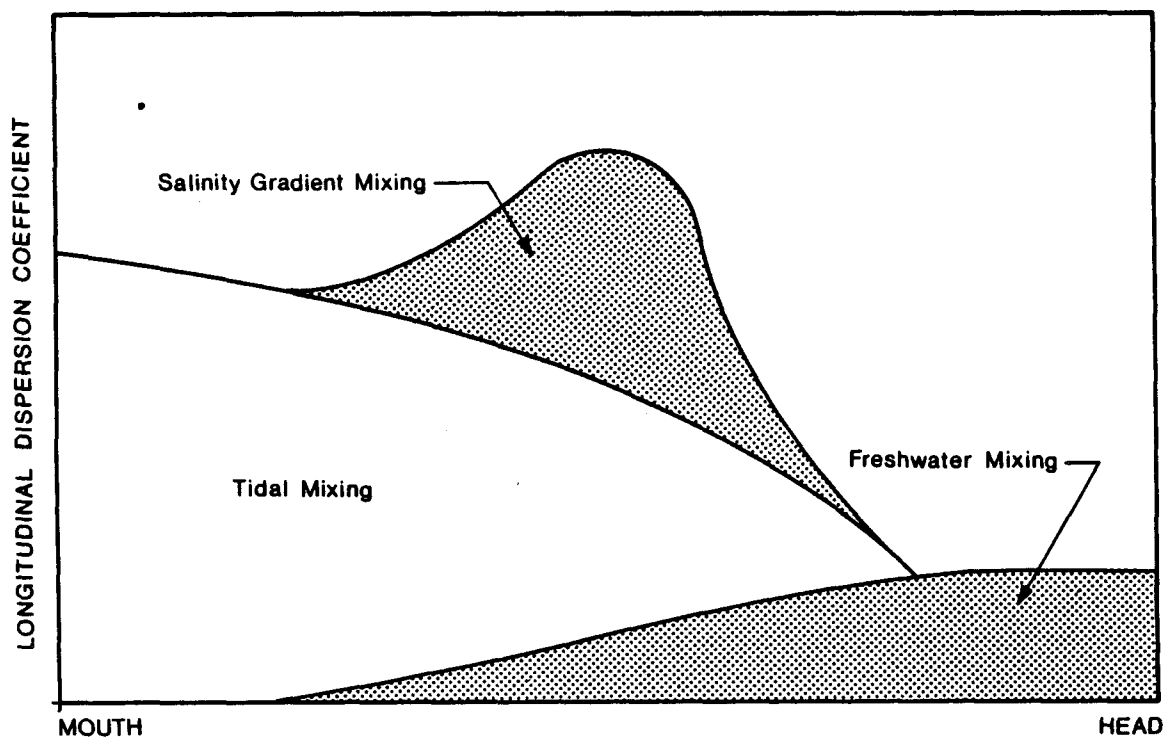


Figure 2-7. Factors contributing to tidally averaged dispersion coefficients in the estuarine environment (modified after Zison et al., 1977).

The expression used is:

$$D_L = C_1 \left( |\bar{u}| + \sigma_u \right) \left( \bar{y} + \sigma_y \right) + C_4 \left( \frac{\Delta S}{\Delta x} \right) \quad (2-47)$$

where  $D_L$  = tidally averaged dispersion coefficient,  $\text{ft}^2/\text{sec}$   
 $C_1$  = tidally-induced mixing coefficient (dimensionless)  
 $\bar{y}$  = tidally averaged depth, ft  
 $|\bar{u}|$  = tidally averaged absolute value of velocity,  $\text{ft}/\text{sec}$   
 $\sigma_u$  = standard deviation of velocity,  $\text{ft}/\text{sec}$   
 $\sigma_y$  = standard deviation of depth, ft  
 $C_4$  = density-induced mixing coefficient,  $\text{ft}^3/\text{sec}/\text{mg}/\text{l-salinity}$   
 $\frac{\Delta S}{\Delta x}$  = salinity gradient,  $\text{mg}/\text{l}/\text{ft}$

The first term on the right side of Equation (2-47) represents mixing brought about by the oscillatory flows associated with the ebbing and flooding of the tide. The second term represents additional mixing when longitudinal salinity gradients are present. It is noted that, in practice the above formulation requires careful calibration using field salinity data due to the high empirical dependency of this relationship.

One common method of experimentally determining the tidally averaged dispersion coefficient is by the "fraction of freshwater method," as explained by Officer (1976). The expression is:

$$D_L = \frac{Rs}{A(ds/dx)} = \frac{R(f-1)}{A(df/dx)} \quad (2-48)$$

where  $D_L$  = tidally averaged dispersion coefficient,  $\text{ft}^2/\text{sec}$

$s$  = mean salinity at a particular location averaged over depth,  $\text{mg/l}$

$A$  = cross-sectional area normal to flow,  $\text{ft}^2$

$R$  = total river runoff flow rate, cfs

$f$  = freshwater fraction =  $\frac{\sigma-s}{\sigma}$ , unitless

$\sigma$  = normal ocean salinity of the coastal water into which the estuary empties,  $\text{mg/l}$

$x$  = distance along estuary axis, ft.

$D_L$  can be calculated at any location within the estuary if the river flow, cross-sectional area, and salinity or freshwater fraction distributions are known.

The above method has certain pitfalls which are pointed out by Ward and Fischer (1971) in their analysis of such an application to the Delaware Estuary. They point out that the use of a dispersion coefficient relationship, i.e., a functional relationship of dispersion to distance, which is also directly related to the measured upstream freshwater inflow, neglects entirely the basic response of the waterbody to variations in freshwater inflow. Ward and Fischer show, for example, that it may take a period of months for the estuary to adjust to a short period change in



freshwater discharge and that any dispersion coefficient relationship based on a simple correlation analyses may be seriously in error.

Hydroscience (1971) has collected values of tidally averaged dispersion coefficients for numerous estuaries, and these values are shown in Table 2-3.

In his book, Officer (1976) reviews studies performed in a number of estuaries throughout the world. He discusses the dispersion coefficients which have been determined, and a summary of values for these estuaries is contained in Table 2-4. Many values were developed using the fraction of freshwater method just discussed. Additional values for the longitudinal dispersion coefficient have been summarized in Fischer et al. (1979).

#### 2.3.4.3 The Lagrangian Method

The models discussed in previous sections of this chapter have all been based on the Eulerian concept of assigning velocities and concentrations to fixed points on a spatial grid. As previously discussed, the fixed grid approach tends to introduce a fictitious "numerical" dispersion into the mass transport solution since the length scale of the diffusion process is somewhat artificially imposed depending on the grid detail. To avoid such a problem, an alternative approach termed the Lagrangian method has been used by Fischer (1972), Wallis (1974), and Spaulding and Pavish (1984) for models of estuaries and tidal waters. Briefly, the Lagrangian method establishes marked volumes of water, distributed along the channel axis, which are moved along the channel at the mean flow velocity. Numerical diffusion is almost entirely eliminated, since there is no allocation of concentrations to specific grid points; rather, the "grid" is a set of moving points which represent the centers of the marked volumes. Longitudinal dispersion between marked volumes can be set according to appropriate empirical or theoretical diffusion behavior (Fischer et al., 1979). The Lagrangian method has been primarily applied to channelized estuaries such as the Suisun Marsh (Fischer, 1977) and Bolinas Lagoon (Fischer, 1972), and more recently has been extended by Spaulding and Pavish (1984) to simulate particulate transport in three dimensions.

TABLE 2-3. TIDALLY AVERAGED DISPERSION COEFFICIENTS FOR SELECTED ESTUARIES  
(from Hydrosience, 1971)

Estuary	Freshwater Inflow (cfs)	Low Flow Net Nontidal Velocity (fps) Head - Mouth	Dispersion Coefficient (mi <sup>2</sup> /day)*	(ft <sup>2</sup> /sec)
Delaware River	2,500	0.12-1.000	5	1610
Hudson River (NY)	5,000	0.037	20	6450
East River (NY)	0	0.0	10	3230
Cooper River (SC)	10,000	0.25	30	9680
Savannah River (GA, SC)	7,000	0.7-0.17	10-20	3230-6450
Lower Raritan River (NJ)	150	0.047-0.029	5	1610
South River (NJ)	23	0.01	5	1610
Houston Ship Channel (TX)	900	0.05	27	8710
Cape Fear River (NC)	1,000	0.48-0.03	2-10	645-3230
Potomac River (VA)	550	0.006-0.003	1-10	320-3230
Compton Creek (NJ)	10	0.10-0.013	1	320
Wappinger and Fishkill Creek (NY)	2	0.004-0.001	0.5-1	160-320

\*1 mi<sup>2</sup>/day = 322.67 ft<sup>2</sup>/sec.

### 2.3.5 Dispersive Transport in Rivers

#### 2.3.5.1 Introduction

Dispersive transport in rivers is typically, but not always, modeled using a one-dimensional equation such as:

$$\frac{\partial C}{\partial t} + \frac{U \partial C}{\partial x} = \frac{\partial}{\partial x} (D_L \frac{\partial C}{\partial x}) \quad (2-49)$$

where C = concentration of solute, mass/length<sup>3</sup>  
U = cross-sectional averaged velocity, length/time  
D<sub>L</sub> = longitudinal dispersion coefficient, length<sup>2</sup>/time  
x = longitudinal coordinate, length  
t = time

TABLE 2-4. TIDALLY AVERAGED DISPERSION COEFFICIENTS  
(FROM OFFICER, 1976)

Estuary	Dispersion Coefficient Range (ft <sup>2</sup> /sec)	Comments
San Francisco Bay, CA Southern Arm Northern Arm	200-2,000 500-20,000	Measurements were made at slack water over a period of one to a few days. The fraction of freshwater method was used. Measurements were taken over three tidal cycles at 25 locations.
Hudson River, NY	4,800-16,000	The dispersion coefficient was derived by assuming $D_L$ to be constant for the reach studied, and that it varied only with flow. A good relationship resulted between $D_L$ and flow, substantiating the assumption.
Narrows of Mercey, UK	1,430-4,000	The fraction of freshwater method was used by taking mean values of salinity over a tidal cycle at different cross sections.
Potomac River, MD	65-650	The dispersion coefficient was found to be a function of distance below the Chain Bridge. Both salinity distribution studies (using the fraction of freshwater method) and dye release studies were used to determine $D_L$ .
Severn Estuary, UK	75-750	Bowden recalculated $D_L$ values originally determined by Stommel, who had used the fraction of freshwater method. Bowden included the freshwater inflows from tributaries, which produced the larger estimates of $D_L$ .
Tay Estuary, UK	530-1,600	The fraction of freshwater method was used. At a given location, $D_L$ was found to vary with freshwater inflow rate.
Thames Estuary, UK	3,640 (high flow) 600-1000 (low flow)	Calculations were performed using the fraction of freshwater method, between 10 and 30 miles below London Bridge.
Yaquina Estuary	650-9,200 (high flow) 140-1,060 (low flow)	The dispersion coefficients for high flow conditions were substantially higher than for low flow conditions, at the same locations. The fraction of freshwater method was used.

Because of the difficulty of accurately solving Equation (2-49) numerically, some researchers (e.g., Jobson, 1980a; Jobson and Rathbun, 1985) have chosen a Lagrangian approach, where the coordinate system is allowed to move with the local stream velocity. Using this approach, Equation (2-49) become:

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial \xi} \left( D_L \frac{\partial C}{\partial \xi} \right) \quad (2-50)$$

where  $\xi = x - \int_0^t U d\tau$

The numerically troublesome advective term does not appear in Equation (2-50). In general, the equation can be solved more easily and with more accuracy than Equation (2-49).

A second method used to simulate dispersive transport in rivers is to consider lateral mixing in addition to longitudinal mixing. A typical form of the two-dimensional equation is:

$$\frac{\partial C}{\partial t} + u(y) \frac{\partial C}{\partial x} = \frac{\partial}{\partial x} \left( \epsilon_x \frac{\partial C}{\partial x} \right) + \frac{\partial}{\partial y} \left( \epsilon_y \frac{\partial C}{\partial y} \right) \quad (2-51)$$

where  $u(y)$  = depth averaged velocity of water, which is a function of  $y$ , and is no longer the cross-sectional averaged velocity, length/time

$\epsilon_x$  = depth averaged longitudinal diffusion coefficient, length<sup>2</sup>/time

$\epsilon_y$  = depth averaged lateral diffusion coefficient, length<sup>2</sup>/time

$y$  = lateral coordinate, length

Note that longitudinal dispersion coefficient,  $D_L$ , in Equation (2-49) is not the same as the longitudinal diffusion coefficient,  $\epsilon_x$ , in Equation (2-51). Typically,  $D_L \gg \epsilon_x$ .

#### 2.3.5.2 Longitudinal Dispersion in Rivers

Fischer (1966, 1967a, 1967b, 1968) has performed much of the earlier research on longitudinal dispersion in natural channels. Prior to Fischer, Taylor (1954) studied dispersion in straight pipes and Elder (1959) studied dispersion in an infinitely wide open channel. More recently Fischer et al. (1979) and Elhadi et al. (1984) have provided a comprehensive review of dispersion processes..

Researchers have shown that Equation (2-49) is valid only after some initial mixing length, often called the Taylor length or convective period. While the convective period has been a topic of active research in the literature (e.g., Fischer, 1967a and b; McQuivey and Keefer, 1976a; Chatwin, 1980), this concept is not embodied in one-dimensional water quality models in general use.

Table 2-5 summarizes references on stream dispersion. The references include information from at least one of the following areas:

- methods to predict  $D_L$ , typically for model applications
- methods to measure  $D_L$  from field data
- data summaries of dispersion coefficients
- approaches used to simulate dispersion in, a non-Fickian manner.

Bansal (1971), Elhadi and Davar (1976), Elhadi et al. (1984) also provide reviews of stream dispersion.

To date, the predictive capabilities of expressions for dispersion coefficients have not been thoroughly tested. However, it is known that the Taylor (1954) or Elder (1959) formulas do not accurately predict dispersion coefficients for natural streams. Glover (1964) found that dispersion coefficients in natural streams were likely to be 10 to 40 times higher than predicted by the Taylor or Elder equations. The lateral variation in stream velocity is the primary reason for the increased dispersion not accounted for by Taylor and Elder. Fischer (1967a) quantified the contribution of the

Reference	Comments
Taylor (1954)	$D_L = 10.1R_p u_*$ ; pipe flow.
Elder (1959)	$D_L = 5.93Hu_*$ ; lateral velocity variation not considered.
Glover (1964)	$D_L = 500Ru_*$ ; natural streams.
Krenkel (1960)	$D_L = 6.4H^{1.24}E^{0.3}$ ; two-dimensional channel. ( $E = USg$ )
Parker (1961)	$D_L = 14.3R^{3/2} \sqrt{2gS}$ ; open channel flow.
Fischer (1967a, 1967b)	$D_L = \frac{U^2}{2} \left( \frac{\sigma_{t2}^2 - \sigma_{t1}^2}{\bar{t}_2 - \bar{t}_1} \right)$ ; concentration variances are measured after an initial period. Long tails may introduce some error. $D_L = \frac{-1}{A} \int_0^b q'(y) dz \int_0^y \frac{1}{\epsilon_y d(y)} dy \int_0^y q'(y) dy.$ <p>where <math>q'(y) = \int_0^{d(y)} (U(y,z) - \bar{u}) dz</math></p> <p>This formula considers the effects of lateral velocity changes.</p> $D_L = \frac{1}{2} \frac{d\sigma^2}{dt}$ $D_L = 0.3u_*^2 \frac{1}{Ru_*}$ ; a simplification of the integral equation above <p>Fischer also discusses another method for determining <math>D_L</math> called the routing procedure.</p>
Elhadi and Davar (1976)	Reviewed many methods to predict $D_L$ . Found $D_L/(Hu_*)$ is not a constant as reported by many researchers.
Fischer (1968)	Field measurements of $D_L$ were made in the Green and Duwamish Rivers.
Bansal (1971)	$\log\left(\frac{KU_s D_L}{U^2 H}\right) = 6.45 - 0.762 \log\left(\frac{\rho U_* H}{\mu}\right)$ $\log\left(\frac{KU_s D_L}{U^2 u_* H}\right) = 6.467 - 0.714 \log\left(\frac{\rho u_* H}{\mu}\right)$
Godfrey and Frederick (1970)	Dispersion tests were summarized in five natural streams; measured dispersion coefficients were from 4 to 35 times greater than predicted by Taylor's (1954) method.
Thackston (1966)	$D_L = 7.25Hu_* \left(\frac{U}{u_*}\right)$ ; 2-D channels.

(continued)

TABLE 2-5. (continued)

Reference	Comments
Thackston and Krenkel (1967)	The limitations of dispersion equations which do not consider lateral velocity variations are discussed. Site specific measurements of $D_L$ are recommended.
Miller and Richardson (1974)	In laboratory experiments, $D_L$ varied from 0.6 ft <sup>2</sup> /sec to 66 ft <sup>2</sup> /sec.
McQuivey and Keefer (1974)	Dispersion coefficient data were reviewed, including hydraulic data, for 17 rivers.
	$D_L = 0.66 \frac{U^3}{\tau} Q_0 / (2S_0 W_0)$
	$D_L = 0.058 \frac{Q_0}{S_0 W_0}$
McQuivey and Keefer (1976b)	Dispersion tests performed in the Mississippi River are summarized.
Liu (1977)	$D_L = \frac{\beta Q^2}{u_* R^3}$
	$\beta \approx 0.18 \left( \frac{\sqrt{gRS}}{U} \right)^{1.5}$
	Summary of $D_L$ values also reported.
Fischer (1975)	$D_L = \frac{0.011 U^2 W^2}{H_u}$
	Liu (1977) shows this is a special case of his formula when $\beta = 0.011$ .
Hays et al. (1966)	Several conceptual models of mass exchange with dead zones are presented and the Fickian Equation is modified to include mass transfer to and from dead zones.
Thackston and Schelle (1970)	Application of Hays et al. (1966) dead zone model to TVA stream data.
Day (1975)	Longitudinal dispersion of fluid particles in small mountain streams in New Zealand was investigated. It was shown that the dispersion coefficient increased with distance and never approached an asymptotic value.
Day and Wood (1976)	Longitudinal dispersion of fluid particles in the Missouri River and in a small mountain stream was investigated. The dispersing particles were shown to behave differently from the Taylor type model. A method to predict dispersion was developed.
Liu and Cheng (1980)	A non-Fickian model is presented to predict stream dispersion.

(continued)

TABLE 2-5. (continued)

Reference	Comments
Sabol and Nordin (1978)	A modified model of stream dispersion is presented that includes the effects of storage along the bed and banks.
Valentine and Wood (1977)	Effects of dead zones on stream dispersion are addressed
Valentine and Wood (1979)	Experimental results are provided to show how dead zones modify longitudinal dispersion.
Rutherford, Taylor, Davies (1980)	A hybrid method is discussed to predict dispersion in the Waikato River, New Zealand.
Beltaos (1980a)	Dispersion processes in streams are reviewed and it is shown that many experimental results do not comply with Fickian dispersion theory. A non-Fickian dispersion model is proposed.
Beltaos (1982)	Dispersion in steep mountain streams is examined.
Bajraktarevic - Dobran (1982)	Fischer's methods are successfully applied to predict dispersion in mountainous streams.
Beer and Young (1983)	Methods are developed to predict dispersion in rivers including the effects of dead zones, using a (J,n,m) model.
Jobson (1980a)	The Fickian Equation is solved with a Lagrangian scheme to avoid lumping numerical dispersion with actual physical dispersion. See Jobson (1980b).
Jobson (1985) and McBride and Rutherford (1984)	Determined that $D_L$ and coefficients for nonconservative water quality constituents could be determined simultaneously during calibration. $D_L$ determined by this method is in good agreement with literature values (Jobson) or match $D_L$ values determined from dye studies (McBride and Rutherford).
Jobson and Rathbun (1985)	Numerical dispersion minimized with a Lagrangian routing procedure that provides more consistent estimates of $D_L$ than the method of moments for pool and riffle streams. Applying this procedure to peak dye concentrations yielded $D_L$ to within 10% of estimates based on the entire concentration-time curves.
(continued)	



TABLE 2-5. (continued)

## Footnotes:

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$A$	= cross-sectional area
$b$	= channel width
$\bar{C}$	= wave velocity
$d(y)$	= depth of water at $y$
$E$	= rate of energy dissipation per unit mass of fluid
$E_y$	= lateral turbulent mixing coefficient
$H$	= stream depth
$K$	= regional dispersion factor
$l$	= lateral distance from location of maximum velocity
$\sigma_x^2$	= variance of distance - concentration curves
$\sigma_{t2}^2, \sigma_{t1}^2$	= variance of time concentration curves
$\bar{t}_2, \bar{t}_1$	= mean times of passage
$\rho$	= mass density of water
$Q_0$	= discharge at steady base flow
$q'(y)$	= integral of velocity deviation on depth
$R$	= hydraulic radius
$R_p$	= pipe radius
$S_0$	= slope of energy gradient at steady base flow
$U$	= mean velocity of flow in reach
$u'$	= deviation of velocity from cross-sectional mean
$U_s$	= mean velocity of flow at sampling point
$u_*$	= shear velocity
$\mu$	= coefficient of viscosity of water
$W_0$	= channel width at steady base flow

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lateral velocity variation on stream dispersion.

A number of the formulas in Table 2-5 are of the type  $D_L/(u_*H) = \text{constant}$ . However, several researchers, including Bansal (1971), Elhadi and Davar (1976), and Beltaos (1978a) have shown that the ratio  $D_L/(u_*H)$  is not a constant. Figure 2-8 shows this ratio can vary by several orders of magnitude.

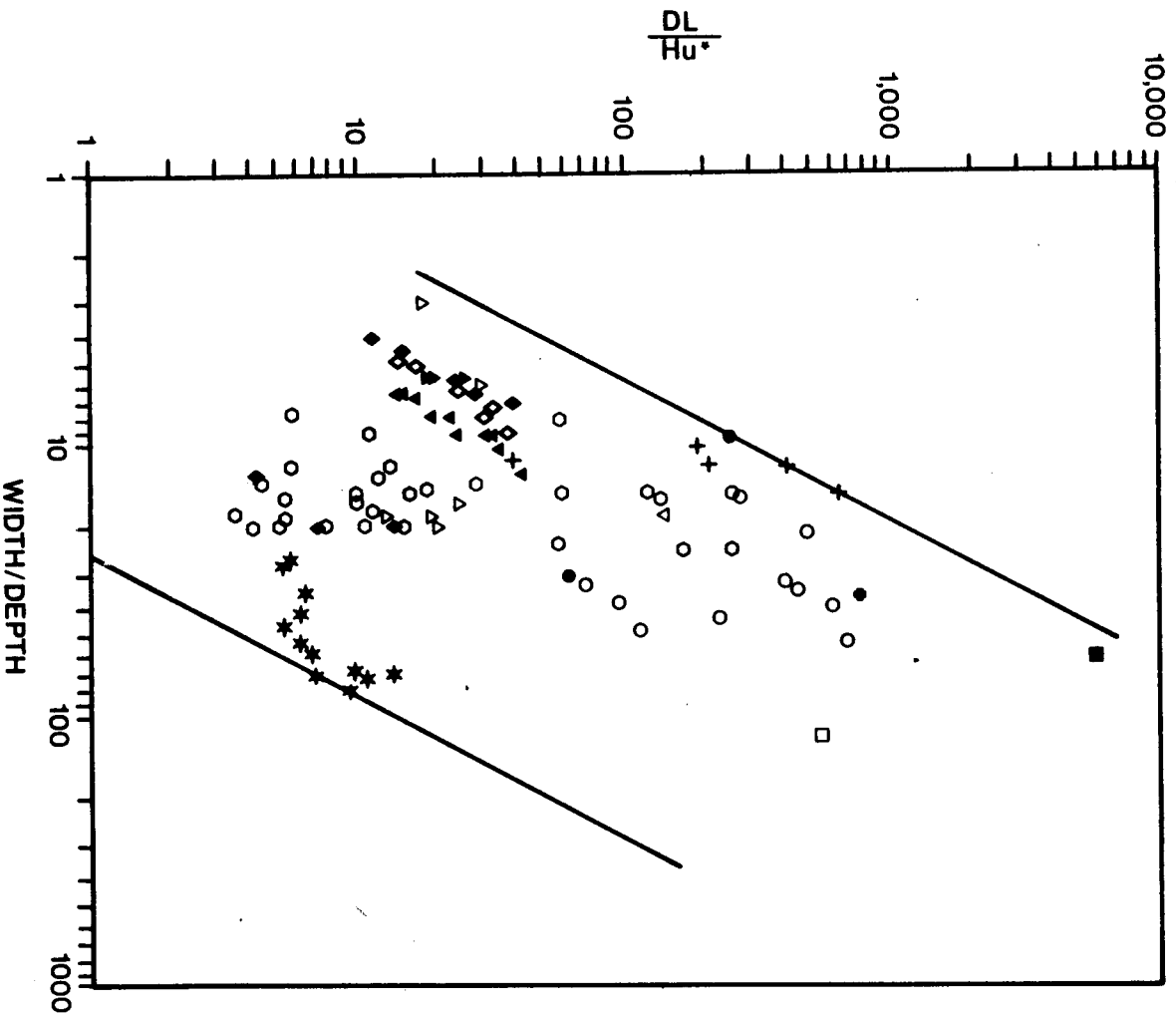
Two widely used methods of predicting the longitudinal dispersion coefficients were developed by Liu (1977) and Fischer (1975) and are shown in Table 2-5. Liu showed that Fischer's method is identical to his own when  $\beta = 0.011$ .

Although numerous researchers (e.g., Sabol and Nordin, 1978) have shown how to include the effects of dead zones on dispersive transport, this refinement does not yet appear to be in general use in water quality models today. In fact, some water quality models do not include dispersion at all (at least physical dispersion; numerical dispersion may be present, depending on the solution technique used).

Dispersion can be neglected in certain circumstances with very little effect on the predicted concentration distributions. Thomann (1973), Li (1972), and Ruthven (1971) have investigated the influence of dispersion. Ruthven gave a particularly simple expression for a pollutant which decays at a rate  $k$ . If

$$\frac{kD_L}{U^2} < \frac{1}{23} = .04$$

then the concentration profile will be affected by no more than 10 percent if dispersion is ignored. Consider, for example, a decaying pollutant with  $k = 0.5/\text{day}$  in a stream where  $U = 1 \text{ fps}$  and  $D_L = 500 \text{ ft}^2/\text{sec}$ . The ratio  $kD_L/U^2 = .003$ , which indicates that dispersion can be ignored. This guideline assumes that the pollutant is being continuously released and conditions are at steady state. The basic presumption is that if the concentration gradient is small enough, the dispersive transport is also small, and



○ Godfrey and Frederick (1970)  
 △ Glover (1964), rectangular flume  
 ▲ Glover (1964), triangular flume  
 □ Glover (1964), South Platte River  
 ◆ Glover (1964), Mohawk River  
 ■ Yotsukura et. al. (1970), Missouri River  
 ● Fischer, Sacramento River (see Sooky, 1969)  
 ▽ Fischer (1968), Green-Duwamish River  
 + Fischer (1967), trapezoidal flume

▼ Smooth, meandering flume<sup>1</sup>  
 ◇ Rough, meandering flume<sup>1</sup>  
 ◆ Smooth, meandering flume<sup>2</sup>  
 ★ Thackston and Schnelle (1969)  
 ○ Hou and Christensen (1976)

<sup>1</sup> Width/radius of curvature = 0.28  
<sup>2</sup> Width/radius of curvature = 0.14

Figure 2-8. Dispersion coefficients in streams (Beltaos, 1978a).

perhaps negligible. On the other hand when pollutants are spilled, concentration gradients are large and dispersion is not negligible.

Thomann (1973) investigated the importance of longitudinal dispersion in rivers that received time variable waste loadings, and therefore produced concentration gradients in the rivers. His results showed that for small rivers, dispersion may be important when the waste loads vary with periods of 7 days or less. For large rivers, dispersion was found to be important whenever the waste load was time-variable.

#### 2.3.5.3 Lateral Dispersion in Rivers

Although two-dimensional water quality models are less widely used in rivers than one-dimensional models, lateral mixing has been the topic of considerable research. Models that simulate lateral mixing are particularly useful in wide rivers where the one dimensional approach may not be applicable. Vertical mixing is rarely simulated in river modeling because the time required for vertical mixing is usually very rapid compared to the time required for lateral mixing. Thermal plumes are an exception.

An example of a model that simulates lateral mixing in rivers is the RIVMIX model of Krishnappan and Lau (1982). The model is particularly useful for delineating mixing zones or regulating the rate of pollutant discharge so that concentrations outside of the mixing zones are limited to allowable values.

When lateral and longitudinal mixing are both simulated, the x and y coordinates are generally assumed to continuously change to be oriented in the longitudinal and transverse directions. Although Equation (2-51) should rigorously contain metric factors (Fukuoka and Sayre, 1973) to account for these continuous changes, modelers typically assume the metric factors are unity.

Lateral mixing coefficients are usually presented in one of the following two forms:

$$\epsilon_y = \alpha H u_* \quad (2-52)$$

or

$$D_y = \frac{\beta Q^2}{W} \quad (2-53)$$

where  $\epsilon_y$  = lateral mixing coefficient, length<sup>2</sup>/time  
 $D_y$  = lateral diffusion factor, length<sup>5</sup>/time<sup>2</sup>  
 $H$  = water depth, length  
 $\alpha, \beta$  = coefficients that vary from river to river  
 $u_*$  = friction velocity, length/time  
 $Q$  = stream flow, length<sup>3</sup>/time  
 $W$  = width of river, length

$D_y$  and  $\epsilon_y$  are related by the following formula:

$$D_y = H^2 U m_x \epsilon_y \quad (2-54)$$

where  $m_x$  = average metric value in x- direction ( $\approx 1$ )

Equation (2-52) is generally the most widely used of the two formulas. Equation (2-53) is used when the two-dimensional convective-diffusion equation is expressed in terms of cumulative discharge (Yotsukura and Cobb, 1972).

Table 2-6 summarizes studies of transverse mixing in streams. Data from the literature are summarized in Tables 2-7 through 2-9. Table 2-9 contains values of  $\beta$  for use in Equation (2-53).

Elhadi et al. (1984) have recently provided a detailed review of lateral mixing in rivers. They concluded that lateral mixing coefficients can be predicted with accuracy only in relatively straight channels.

TABLE 2-6. SUMMARY OF STUDIES OF TRANSVERSE MIXING IN STREAMS

Reference	Comments
Okoye (1970)	This study presented a detailed analysis of laboratory experiments of lateral mixing.
Prych (1970)	This study detailed the effects of density differences on lateral mixing.
Yotsukura, Fischer, Sayre (1970)	A lateral dispersion coefficient of $1.3 \text{ ft}^2/\text{sec}$ was determined for the Missouri River.
Yotsukura and Cobb (1972)	Studies of lateral mixing were performed on the South River, Atrisco Feeder Canal, Bernardo Conveyance Channel, and the Missouri River.
Holley (1975)	A two-dimensional model of contaminant transport in rivers was developed and applied to the Missouri and Clinch Rivers. $\epsilon_y$ was experimentally determined using $\epsilon_y = \frac{U}{2} \frac{d\sigma_y^2}{dx}$
Holley and Abraham (1973)	Transverse dispersion measurements were made in the Waal and IJssel Rivers, Holland. The change of moments method was used.
Yotsukura and Sayre (1976)	Transverse cumulative discharge was used as an independent variable replacing transverse distance in the 2-D mass transport equation.
Shen (1978)	The approach of Yotsukura and Sayre (1976) was extended to include transient mixing.
Lau and Krishnappan (1981)	Field data for transverse mixing coefficients were summarized. A further extension of the approach of Yotsukura and Sayre was made. Values of $\epsilon_y/(u_*H)$ were found to depend on depth/width ratios.
Somlyódy (1982)	Tracer studies were performed in five streams to predict lateral mixing coefficients. A numerical model used in the study was an extension of the work of Yotsukura and Sayre (1976).
Gowda (1978)	Transverse mixing coefficients were measured in the Grand River.
Mescal and Warnock (1978)	A study of lateral mixing in the Ottawa River produced the expression $\epsilon_y = 0.043HU$ .
Benedict (1978)	This study reviewed various mixing expressions.
Henry and Foree (1979)	An approximate method of two-dimensional dispersion modeling was presented.
Beltaos (1980)	Transverse mixing characteristics of three rivers in Alberta, Canada were documented by tracer tests for open water and ice covered flow conditions.
Cotton and West (1980)	Rhodamine WT dye was used to determine the transverse diffusion coefficient on a straight reach of an open channel.
Holley and Nerat (1983)	Inclusion of secondary mixing as part of a lateral diffusion coefficient was concluded to have a limited physical basis.
Demetracopoulous and Stefan (1983)	Transverse mixing was studied in wide and shallow rivers using heated discharge as a tracer. A modified method of moments was developed to compute transverse mixing coefficients.
Webel and Schatzmann (1984)	An experimental study was conducted to investigate variations in transverse mixing coefficients in straight, rectangular channels. $\epsilon_y/(u_*H)$ was found to be constant.

$\epsilon_y$  = lateral mixing coefficient  
 $U$  = cross-sectional average velocity  
 $\sigma_y^2$  = variance of concentration in y-direction  
 $u_*$  = shear velocity  
 $H$  = depth

TABLE 2-7. TRANSVERSE MIXING COEFFICIENTS IN NATURAL STREAMS AND CHANNELS  
(FROM BELTAOS, 1978a)

Source	Channel and Description	W (m)	$\frac{W}{R}$	U (m/s)	f	$\epsilon_y/(Hu_*)$	Comments
Glover 1964	Columbia River	305	100	1.35	.034	.74	Test results and analysis approximate
Yotsukura et al., 1970	Missouri River, two mild alternating bends	183	68.7	1.74	.014	.60	Flow distribution available at only two cross sections
Yotsukura and Cobb, 1972	South River, few mild bends	18.2	46.2	.21	.284	.30	Analysis by streamtube method
Sayre and Yeh, 1973	Missouri River, sinuous, severe bends	234	59.1	1.98	.015	3.30	Analysis by numerical and analytical methods. Periodical variation of $\epsilon_y$ detected; average value indicated here
Engmann and Kellerhals, 1974	Lesser Slave River, irregular, almost contorted meander, no bars; sinuosity = 2.0	43.0	17.0	.65	.045	.33	Effects of transverse advection lumped together with transverse dispersion. Reanalysis of ice covered data by streamtube method gave $\epsilon_y/Ru_* = .16$
Meyer, 1977	Mobile River, mostly straight, one mild curve	430	87.2	.30	.028	7.20	Steady-state condition unlikely
Krishnappan & Lau, 1977	Meandering laboratory flume with "equilibrium bed". Planview sinuoidal. Meander wavelength=2 $\pi$ W=1.88m	.30	10.5	.26	.162	-	Evaluation of $\epsilon_y$ by a numerical simulation method. Use of constant $C_y$ gave more consistent results than laterally variable values of $\epsilon_y$ .
		.30	15.9	.27	.105	-	
		.30	7.6	.31	.163	-	
		.30	10.2	.30	.208	-	
		.30	9.0	.28	.271	-	
		.30	11.6	.23	.156	-	
Beltaos, 1978b	Athabasca River below Fort McMurray, straight with occasional islands, bars; sinuosity=1.0	.30	10.0	.32	.101	-	
		373	170	.95	.028	.75	
Beltaos, 1978b	Athabasca River below Athabasca, irregular meanders with occasional bars, islands; sinuosity=1.2	320	156	.86	.067	.41	Slug-injection tests; analysis by streamtube method applied to dosage (see also Beltaos 1975)
Beltaos, 1978b	Beaver River near Cold Lake, regular meanders, point bars and large dunes, sinuosity=1.3	42.7	44.6	.50	.062	1.0	Steady-state concentration tests. Analysis by stream-tube method.
Beltaos (unpublished)	North Saskatchewan River below Edmonton, nearly straight, few, very mild bends with occasional bars, islands; sinuosity=1.0	213	137	.58	.152	.25	By steady-state concentration and slug-injection tests. Analysis by streamtube and numerical methods respectively
Beltaos (unpublished)	Bow River at Calgary, sinuous with frequent islands; mid-channel bars diagonal bars, sinuosity=1.1	104	104	1.05	.143	.61	

A = amplitude of meanders  
f = friction factor  
R = hydraulic radius  
U = cross-sectionally averaged velocity  
W = width  
H = depth  
 $\epsilon_y$  = lateral mixing coefficient

TABLE 2-8. SUMMARY OF FIELD DATA FOR TRANSVERSE DISPERSION COEFFICIENTS  
(LAU AND KRISHNAPPEN, 1981)

Data Source	Width, in meters	W/H	Average velocity in meters per second	Shear Velocity in meters per second	Friction factor	Dispersion Coefficient, E <sub>y</sub> , in meters squared per second	$E_y/u_*H$	$E_y/u_*H$	Sinuosity S
Yotsukura and Cobb (1972) Missouri River near Blair	183.0	66.7	1.74	0.073	0.014	0.101	$7.5 \times 10^{-3}$	0.50	1.1
Yotsukura and Cobb (1972) South River	18.3	46.2	0.18	0.040	0.220	0.0046	$6.3 \times 10^{-3}$	0.29	1.0 <sup>a</sup>
Yotsukura and Cobb (1972) Aristo Feeder Canal	18.3	27.3	0.67	0.062	0.069	0.0093	$8.2 \times 10^{-3}$	0.22	1.0 <sup>a</sup>
Yotsukura and Cobb (1972) Bernado Conveyance Channel	20.1	28.7	1.25	0.061	0.020	0.013	$10.6 \times 10^{-3}$	0.30	1.0 <sup>a</sup>
Beltaos (1978a), Athabasca below Fort McMurray	373.0	170.0	0.95	0.056	0.028	0.092	$4.4 \times 10^{-3}$	0.75	1.0 <sup>a</sup>
Beltaos (1978a), Athabasca River below Athabasca	320.0	156.0	0.86	0.079	0.067	0.066	$2.6 \times 10^{-3}$	0.41	1.2
Beltaos (1978a), North Saskatchewan River below Edmonton	213.0	137.0	0.58	0.080	0.152	0.031	$1.8 \times 10^{-3}$	0.25	1.0 <sup>a</sup>
Beltaos (1978b), Bow River at Calgary	104.0	104.0	1.05	0.139	0.143	0.085	$5.9 \times 10^{-3}$	0.61	1.1
Beltaos (1978b), Beaver River near Cold Lake	42.7	44.6	0.50	0.044	0.062	0.042	$22.4 \times 10^{-3}$	1.00	1.3
Sayre and Yeh (1975) Missouri River below Cooper Generation Station	234.0	59.1	1.98	0.085	0.015	1.110	$55.8 \times 10^{-3}$	3.30	2.1
Lau and Krishanppan (1977) Grand River below Kitchener	59.2	117.0	0.35	0.069	0.314	0.009	$2.2 \times 10^{-3}$	0.26	1.1



TABLE 2-9. SUMMARY OF NONDIMENSIONAL DIFFUSION FACTORS IN NATURAL STREAMS  
(FROM GOWDA, 1984)

Source of data	Salient features	Discharge, in cubic meters per second	Mean width, in meters	Mean depth, in meters	Mean velocity in meters per second	Nondimensional diffusion factor, $\beta$
Hamdy and Kinhead (1979) St. Clair River	12.0 km straight stretch with an island	6,800.00	819.3	10.00	0.83	$5.9 \times 10^{-4}$
Glover (1964) Columbia River near Richland	0.11 km stretch with a gradual S-curve	1,235.30	304.8	3.00	1.35	$4.7 \times 10^{-4}$
Holley and Abraham (1973) Waal River	10.0 km straight stretch	1,027.75	266.1	4.70	0.82	$5.3 \times 10^{-4}$
Yotsukura and Cobb (1972) Missouri River near Blair	10.0 km stretch with mild alter- nating curve	965.60	183.0	2.74	1.74	$6.6 \times 10^{-4}$
Beltaos (1980b) Athabasca River below Fort McMurray	17.6 km stretch with occasional bars and islands	776.00	373.0	2.20	0.95	$7.8 \times 10^{-4}$
Beltaos (1980b) Athabasca River below Athabasca	17.0 km stretch with irregular meanders, occa- sional bars and islands	566.00	320.0	2.05	0.86	$8.4 \times 10^{-4}$
Holly and Abraham (1973) IJssel River	8.6 km stretch with three al- ternating bends	269.75	69.5	4.00	0.97	$23.0 \times 10^{-4}$
Beltaos (1980b) Beaver River near Cold Lake	1.5 km stretch with regular meanders, point bars and large dunes	20.5	42.7	0.96	0.50	$41.0 \times 10^{-4}$
Yotsukura and Cobb (1972) Bernardo Conve- yance Channel	2.0 km straight stretch	17.75	20.1	0.70	1.25	$81.0 \times 10^{-4}$
Gowda (1980) Grand River below Waterloo	3.4 km stretch with two alter- nating curves	12.54	57.3	0.56	0.39	$10.0 \times 10^{-4}$
Yotsukura and Cobb (1972) Atrisco Feeder Canal near Bernalillo	2.0 km straight stretch with a channel of nearly uniform cross- section	7.42	18.3	0.67	0.67	$13.0 \times 10^{-4}$
Yotsukura and Cobb (1972) South River near the Town of Waynesboro	0.4 km stretch with a few very slight bends	1.53	18.2	0.38	0.21	$25.0 \times 10^{-4}$
Gowda (1980) Boyne River below Alliston	0.2 km straight stretch	0.82	8.85	0.43	0.22	$25.0 \times 10^{-4}$

Notes:

$$\beta = \frac{D_y W}{Q^2}$$

$$D_y = H^2 U m_x c_y$$

W = channel width

Q = flow rate

H = depth

U = velocity

$m_x$  = average value of matrix ( $\approx 1$ ) in x-direction

### 2.3.6 Summary

The previous sections have provided a brief review on the treatment of dispersive transport in water quality models. This has included a discussion of vertical dispersion in lakes and estuaries, and horizontal (lateral and longitudinal) dispersion in lakes, estuaries, and rivers. It is readily seen that a wide variety of numerical formulations for dispersion exist in the literature. Formulations for dispersion coefficients tend to be model-dependent and are all based to some extent on general lack of a complete understanding of the highly complex turbulence induced mixing processes which exist in natural water bodies. In all cases, due to this model and empirical dependence, it is desirable to include a careful calibration and/or verification exercise using on-site field data for any water quality modeling application.

## 2.4 SURFACE HEAT BUDGET

The total heat budget for a water body includes the effects of inflows (rivers, discharges), outflows, heat generated by chemical-biological reactions, heat exchange with the stream bed, and atmospheric heat exchange at the water surface. In all practicality, however, the dominant process controlling the heat budget is the atmospheric heat exchange, which is the focus of the following paragraphs. In addition, however, it is also important to include the proper boundary conditions for advective exchange (e.g., rivers, thermal discharges, or tidal flows) when the relative source temperature and rate of advective exchange is great enough to affect the temperature distribution of the water body.

The transfer of energy which occurs at the air-water interface is generally handled in one of two ways in river, lake, and estuary models. A simplified approach is to input temperature values directly and avoid a more complete formulation of the energy transfer phenomena. This approach is most often applied to those aquatic systems where the temperature can be readily measured. Alternatively, and quite conveniently, the various energy transfer phenomena which occur at the air-water interface can be considered in a heat budget formulation.

In a complete atmospheric heat budget formulation, the net external heat flux,  $H$ , is most often formulated as an algebraic sum of several component energy fluxes (e.g., Baca and Arnett, 1976; U.S. Army Corps of Engineers, 1974; Thomann *et al.*, 1975; Edinger and Buchak, 1978; Ryan and Harleman, 1973; TVA, 1972). A typical expression is given as:

$$H = Q_s - Q_{sr} + Q_a - Q_{ar} - Q_{br} - Q_e \pm Q_c \quad (2-55)$$

where  $H$  = net surface heat flux

$\uparrow Q_s$  = shortwave radiation incident to water surface, 30 to 300 kcal/m<sup>2</sup>/hr

$\uparrow Q_{sr}$  = reflected short wave radiation, 5 to 25 kcal/m<sup>2</sup>/hr

$\uparrow Q_a$  = incoming long wave radiation from the atmosphere, 225 to 360 Kcal/m<sup>2</sup>/hr

$\uparrow Q_{ar}$  = reflected long wave radiation, 5 to 15 kcal/m<sup>2</sup>/hr

$\uparrow Q_{br}$  = back radiation emitted by the body of water, 220 to 345 kcal/m<sup>2</sup>/hr

$\uparrow Q_e$  = energy utilized by evaporation, 25 to 900 kcal/m<sup>2</sup>/hr

$\updownarrow Q_c$  = energy convected to or from the body of water, -35 to 50 kcal/m<sup>2</sup>/hr at the surface

NOTE: The magnitudes are typical for middle latitudes of the United States. The arrows indicate if energy is coming into the system ( $\downarrow$ ), out of the system ( $\uparrow$ ), or both ( $\updownarrow$ ).

These flux components can be calculated within the models from semi-theoretical relations, empirical equations, and basic meteorological data. Depending on the algebraic formulation used for the net heat flux term and the particular empirical expressions chosen for each component, all or some of the following meteorological data may be required: atmospheric pressure, cloud cover, wind speed and direction, wet and dry bulb air temperatures, dew point temperature, short wave solar radiation, relative humidity, water temperature, latitude, and longitude.

Estimation of the various heat flux components has been the subject of many theoretical and experimental studies in the late 1960's and early

1970's. Most of the derived equations rely heavily on empirical coefficients. These formulations have been reviewed extensively by the Tennessee Valley Authority (1972), Ryan and Harleman (1973), Edinger et al. (1974), and Paily et al. (1974). A summary of the most commonly used formulations in water quality models is given in the following sections.

#### 2.4.1 Measurement Units

The measurement units in surface heat transfer calculations do not follow any consistent units system. For heat flux, the English system units are BTU/ft<sup>2</sup>/day. In the metric system, the units are either Kcal/m<sup>2</sup>/hr or watt/m<sup>2</sup> (1 watt = 1 joule/sec). The Langley (abbreviated Ly), equal to 1 cal/cm<sup>2</sup>, also persists in usage. The following conversions are useful in this section:

1 BTU/ft <sup>2</sup> /day	= 0.131 watt/m <sup>2</sup>	= 0.271 Ly/day	= 0.113 kcal/m <sup>2</sup> /hr
1 watt/m <sup>2</sup>	= 7.61 BTU/ft <sup>2</sup> /day	= 2.07 Ly/day	= 0.86 kcal/m <sup>2</sup> /hr
1 Ly/day	= 0.483 watt/m <sup>2</sup>	= 3.69 BTU/ft <sup>2</sup> /day	= 0.42 kcal/m <sup>2</sup> /hr
1 kcal/m <sup>2</sup> /hr	= 1.16 watt/m <sup>2</sup>	= 2.40 Ly/day	= 8.85 BTU/ft <sup>2</sup> /day
1 kilopascal	= 10 mb	= 7.69 mm Hg	= 0.303 in Hg
1 mb	= 0.1 kilopascal	= 0.769 mm Hg	= 0.03 in Hg
1 mm Hg	= 1.3 mb	= 0.13 kilopascal	= 0.039 in Hg
1 in Hg	= 33.0 mb	= 25.4 mm Hg	= 3.3 kilopascal

#### 2.4.2 Net short wave Solar Radiation, Q<sub>sn</sub>

Net short wave solar radiation is the difference between the incident and reflected solar radiations ( $Q_s - Q_{sr}$ ). Techniques are available and described in the aforementioned references to estimate these fluxes as a function of meteorological data. However, in order to account for the reflection, scattering, and absorption incurred by the radiation through interaction with gases, water vapor, clouds, and dust particles, a great deal of empiricism is involved and the necessary data are relatively extensive if precision is desired.

One of the most common simplified formulations for net short wave solar radiation (Anderson, 1954; Ryan and Harleman, 1973) is expressed as:

$$Q_{sn} = Q_s - Q_{sr} \approx 0.94 Q_{sc} (1 - 0.65C^2) \quad (2-56)$$

where  $Q_{sc}$  = clear sky solar radiation, kcal/m<sup>2</sup>/hr  
 $C$  = fraction of sky covered by clouds

As reported by Shanahan (1984), Equation (2-56) is an approximation in that it assumes average reflectance at the water surface and employs clear sky solar radiation. In certain circumstances atmospheric attenuation mechanisms are much greater than normal, even under cloudless conditions. For such situations, the more complex formulae described by TVA (1972) are required.

A number of methods are available for estimating the clear sky solar radiation. TVA (1972) presents a formula for  $Q_{sc}$  as a function of the geographical location, time of year, and hour of the day. Thackston (1974) and Thompson (1975) report methods for calculating daily average values of solar radiation as a function of latitude, longitude, month, and sky cover. Hamon et al. (1954) have graphed the daily average insolation as a function of latitude, day of year and percent of possible hours of sunshine, and is given in Figure 2-9.

Lombardo (1972) represents the net short wave solar radiation,  $Q_{sn}$  (langley/day), with the following expression:

$$Q_{sn} = (1-R) Q_s \quad (2-57)$$

where  $Q_s$  = short wave radiation at the surface (langley/day)  
 $R$  = reflectivity of water = 0.03, or alternately:  
 $R = Aa^B$  (A,B given below in Table 2-10)  
 $a$  = sun's altitude in degrees

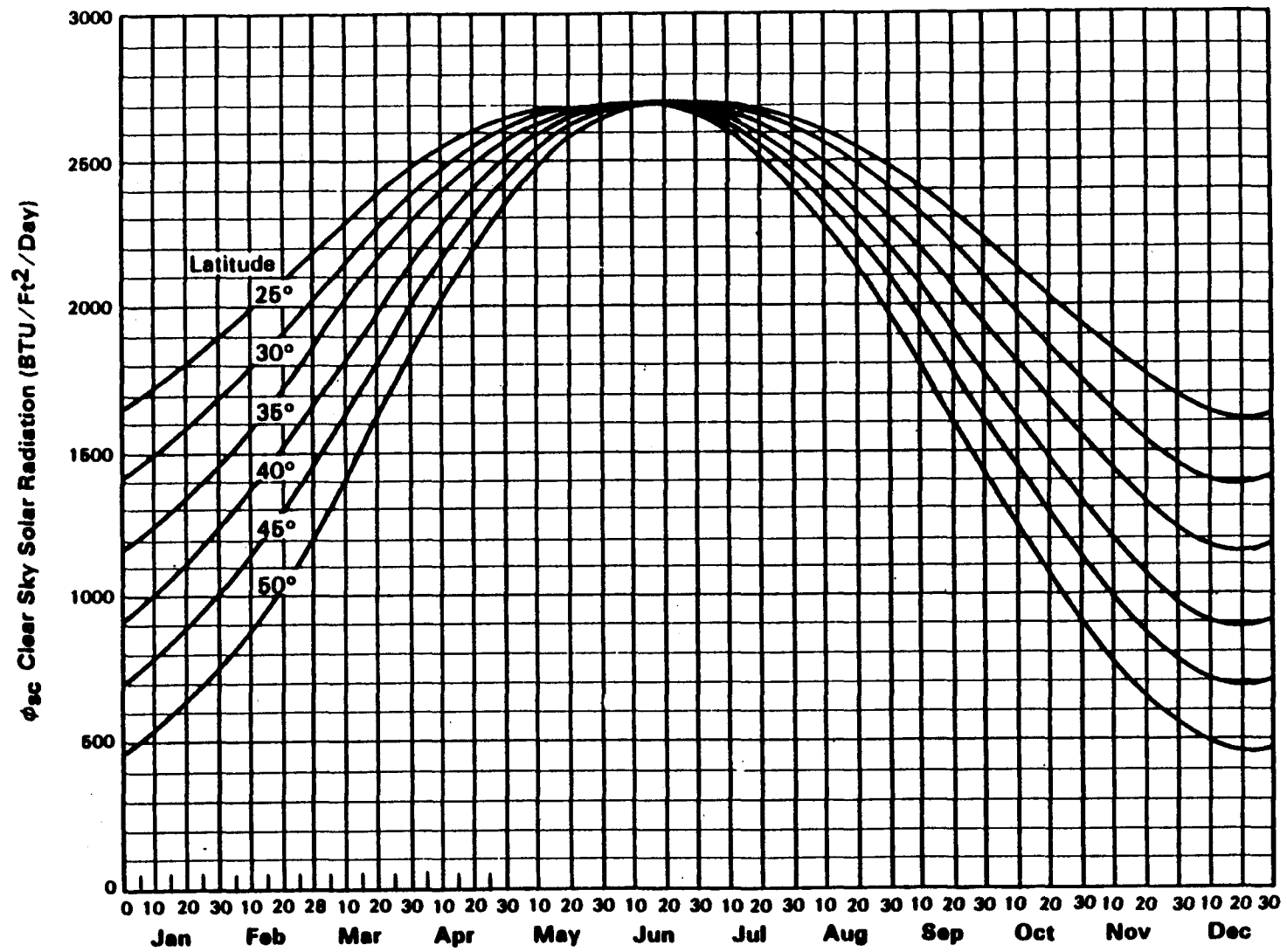


Figure 2-9. Clear sky solar radiation according to Hamon, Weiss and Wilson (1954)

TABLE 2-10. VALUES FOR SHORT WAVE RADIATION COEFFICIENTS A AND B  
(LOMBARDO, 1972)

Cloudiness	Clear	Scattered	Broken	Overcast
A	1.18	2.20	0.95	0.35
B	-0.77	-0.97	-0.75	-0.45

The WQRRS model by the U.S. Army Corps of Engineers (1974) considers the net short wave solar radiation rate ( $Q_s - Q_{sr}$ ) as a function of sun angle, cloudiness, and the level of particulates in the atmosphere. Chen and Orlob, as reported by Lombardo (1973), determine the net short wave solar radiation by considering absorption and scattering in the atmosphere.

A final important note on calculation of the net short wave solar radiation regards the effects of shading from trees and banks primarily on stream systems or rivers with steep banks. Shading can significantly reduce the incoming solar radiation to the water surface, resulting in water temperatures much lower than those occurring in unobstructed areas. Jobson and Keefer (1979) present a method to account for the reduction of incoming solar radiation by prescribing geometric relations of vertical obstruction heights and stream widths for each subreach of their model of the Chattahoochee River.

#### 2.4.3 Net Atmospheric Radiation, $Q_{an}$

The atmospheric radiation is characterized by much longer wavelengths than solar radiation since the major emitting elements are water vapor, carbon dioxide, and ozone. The approach generally adopted to compute this flux involves the empirical determination of an overall atmospheric emissivity and the use of the Stephan-Boltzman law (Ryan and Harleman, 1973). The formula by Swinbank (1963) has been adopted by many investigators for use in various water quality models (e.g., U.S. Army Corps of Engineers, 1974; Chen and Orlob, 1975; Brocard and Harleman, 1976). This

formula was believed to give reliable values of the atmospheric radiation within a probable error to +5 percent. Swinbank's formula is:

$$Q_{an} = Q_a - Q_{ar} = 1.16 \times 10^{-13} (1 + 0.17C^2) (T_a + 460)^6 \quad (2-58)$$

where  $Q_{an}$  = net long wave atmospheric radiation, BTU/ft<sup>2</sup>/day

$C$  = cloud cover, fraction

$T_a$  = dry bulb air temperature, °F

A recent investigation by Hatfield et al. (1983) has found that the formula by Brunt (1932) gives more accurate results over a range of latitudes of 26°13'N to 47°45'N and an elevation range of -30m to + 3,342m. Brunt's formula is:

$$Q_{an} = 2.05 \times 10^{-8} (1 + 0.17C^2) (T_a + 460)^4 (1 + 0.149 \sqrt{e_2}) \quad (2-59)$$

where  $Q_{an}$  = net long wave atmospheric radiation, BTU/ft<sup>2</sup>/day

$e_2$  = the air vapor pressure 2 meters above the water surface, mm Hg

$T_a$  = air temperature 2 meters above the water surface, °F

#### 2.4.4 Long Wave Back Radiation, $Q_{br}$

The long wave back radiation from the water surface is usually the largest of all the fluxes in the heat budget (Ryan and Harleman, 1973). Since the emissivity of a water surface (0.97) is known with good precision, this flux can be determined with accuracy as a function of the water surface temperature:

$$Q_{br} = 0.97 \sigma T_s^4 \quad (2-60)$$

where  $Q_{br}$  = long wave back radiation, cal/m<sup>2</sup>/sec

$T_s$  = surface water temperature, °K

$\sigma$  = Stefan-Boltzman constant =  $1.357 \times 10^{-8}$ , cal/m<sup>2</sup>/sec/°K<sup>4</sup>



The U.S. Army Corps of Engineers (1974) uses the following linearization of Equation (2-60) to express the back radiation emitted by the water body:

$$Q_{br} = 73.6 + 1.17 T \quad (2-61)$$

where  $T$  = water temperature,  $^{\circ}\text{C}$

In the range of  $0^{\circ}$  to  $30^{\circ}\text{C}$ , this linear function has a maximum error of less than 2.1 percent relative to Equation (2-60).

#### 2.4.5 Evaporative Heat Flux, $Q_e$

Evaporative heat loss occurs as a result of the change of state of water from a liquid to vapor, requiring sacrifice of the latent heat of vaporization. The basic formulation used in all heat budget formulations (e.g., Ryan and Harleman, 1973; U.S. Army Corps of Engineers, 1974; Chen and Orlob, 1975; Lombardo, 1972) is:

$$Q_e = \rho L_w E \quad (2-62)$$

where  $Q_e$  = heat loss due to evaporation,  $\text{kcal/m}^2/\text{sec}$

$\rho$  = fluid density,  $\text{kg/m}^3$

$L_w$  = latent heat of vaporization,  $\text{kcal/kg}$

or  $L_w = 597 - 0.57 T_s$

$E$  = evaporation rate,  $\text{m/sec}$

$T_s$  = surface water temperature,  $^{\circ}\text{C}$

The general expression for evaporation from a natural water surface is usually written as:

$$E = (a + bW) (e_s - e_a) \quad (2-63)$$

where  $a, b$  = empirical coefficients

- W = wind speed at some specified elevation above water surface, m/sec
- $e_s$  = saturation vapor pressure at the surface water temperature, mb
- $e_a$  = vapor pressure of the overlying atmosphere, mb

Various approaches have been used to evaluate the above expression. In a very simplified approach, the empirical coefficient,  $a$ , has often been taken to be zero, while  $b$  ranges from  $1 \times 10^{-9}$  to  $5 \times 10^{-9}$  (U.S. Army Corps of Engineers, 1974). The value of  $e_s$  is a nonlinear function of the surface water temperature. However  $e_s$  can be estimated in a piecewise linear fashion as follows:

$$e_s = a_i + \beta_i T_s \quad (2-64)$$

where  $a_i, \beta_i$  = empirical coefficients with values as given in Table 2-11.

$T_s$  = surface water temperature,  $^{\circ}\text{C}$

TABLE 2-11. VALUES FOR EMPIRICAL COEFFICIENTS

Temperature Range, $^{\circ}\text{C}$	$a_i$	$\beta_i$
0-1	6.05	0.522
5-10	5.10	0.710
10-15	2.65	0.954
15-20	-2.04	1.265
20-25	-9.94	1.659
25-30	-22.29	2.151
30-35	-40.63	2.761
35-40	-66.90	3.511

A more convenient formula for the saturation vapor pressure,  $e_s$ , is presented by Thackston (1974) as follows:

$$e_s = \exp \left[ 17.62 - 9501/(T_s + 460) \right] \quad (2-65)$$

where  $e_s$  = saturation vapor pressure at the surface water temperature,  
in Hg

$T_s$  = water temperature, °F

The standard error of prediction of Equation (2-55) is reported by Thackston (1974) to be 0.00335.

A large number of evaporation formula exist for a natural water surface, as demonstrated in Table 2-12 (Ryan and Harleman, 1973). Detailed comparisons of these formulae by the above authors showed that the discrepancies between these formulae were not significant. Both Ryan and Harleman (1973), and TVA (1968) recommend the use of the Lake Hefner evaporation formula developed by Marciano and Harbeck (1954), which has the best data base, and has been shown to perform satisfactorily for other water bodies. The Lake Hefner formula is written as:

$$Q_e = 17 W_2 (e_s - e_2) \quad (2-66)$$

where  $Q_e$  = heat loss due to evaporation, BTU/ft<sup>2</sup>/day

$W_2$  = wind speed at 2 meters above surface, mph

$e_s$  = saturated vapor pressure at the surface water temperature,  
mm Hg

$e_2$  = vapor pressure at 2 meters above surface, mm Hg

It is important to note that the Lake Hefner formula was developed for lakes and may not be universally valid for streams or open channels due to physical blockage of the wind by trees, banks, etc.; and due to differences in the surface turbulence which affects the liquid film aspects of evaporation (McCutcheon, 1982). Jobson developed a modified evaporation formula which was used in temperature modeling of the San Diego Aqueduct (Jobson, 1980) and the Chattahoochee River (Jobson and Keefer, 1981). This formula is written as:

$$E = 3.01 + 1.13 W (e_s - e_a) \quad (2-67)$$

TABLE 2-12. EVAPORATION FORMULA FOR LAKES AND RESERVOIRS  
(RYAN AND HARLEMAN, 1973)

Name	Formula in Original Form	Units*	Observation Levels	Time Increments	Water Body	Formula at sea-level Meas. Ht. Spec. Units BTU/ft <sup>2</sup> /day mph, mm Hg	Remarks
Lake Hefner	$E = 6.25 \cdot 10^{-4} W_8 (e_s - e_8)$	cm/3 hr knots mb	8m-wind 8m- $e_a$	3 hrs Day	Lake Hefner Oklahoma 2587 acres	$12.4 W_8 (e_s - e_8)$ $17.2 W_2 (e_s - e_2)$	Good agreement with Lake Mead, Lake Eucumbene, Russian Lakes.
Kohler	$E = .00304 W_4 (e_s - e_2)$	in./day miles/day in. Hg	4m-wind 2m- $e_a$	Day	Lake Hefner Oklahoma 2587 acres	$15.9 W_4 (e_s - e_2)$ $17.5 W_2 (e_s - e_2)$	Essentially the same as the Lake Hefner Formula.
Zaykov	$E = [.15 + .108 W_2] (e_s - e_2)$	mm/day m/s mb	2m-wind 2m- $e_a$		Ponds and small reservoirs	$(43 + 14 W_2) (e_s - e_2)$	Based on Russian experience. Recommended by Shulyakovskiy
Meyer	$E = 10(1 + .1 W_8) (e_s - e_8)$	in./month mph in. Hg	25 ft-wind 25 ft- $e_a$	Monthly	Small lakes and reservoirs	$(73 + 7.3 W_3) (e_s - e_8)$ $(80 + 10 W_2) (e_s - e_2)$	$e_a$ is obtained daily from mean morning and evening measurements of $T_a$ , $R_H$ . Increase constants by 10% if average of maximum and minimum used.
Morton	$E = (300 + 50 W) (e_s - e_a) / p$	in./month mph in. Hg	8m-wind 2m- $e_a$	Monthly	Class A pan	$(73.5 + 12.2 W_8) (e_s - e_2)$ $(73.5 + 14.7 W_2) (e_s - e_2)$	Data from meteorological stations. Measurement heights assumed.
Rohwer	$E = .771 [1.465 - .01868] \times [.44 + .118 W] (e_s - e_a)$  where B=atmos. press.	in./day mph in. Hg	0.5-1 ft-wind 1 inch- $e_a$	Daily	Pans 85 ft diameter tank 1300 acre Reservoir	$(67 + 10 W_2) (e_s - e_2)$	Extensive pan measurements using several types of pans. Correlated with tank reservoir data.

\*For each formula, the units are for evaporation rate, wind speed, and vapor pressure.

where  $E$  is in mm/day

$W$  = wind speed at some specified elevation above the water surface, m/sec

$e_a$  = vapor pressure at the same elevation as the wind, kilopascals

$e_s$  = saturation vapor pressure at the water surface temperature, kilopascals

It is noted that the wind speed function of Equation (2-67) was reduced by 30 percent during calibration of the temperature model for the Chattahoochee River (McCutcheon, 1982). The original Equation (2-67) was developed for the San Diego Aqueduct which represented substantially different climatic and exposure conditions than for the Chattahoochee River. McCutcheon (1982) notes that the wind speed function is a catchall term that must compensate for a number of difficulties which include, in part:

- Numerical dispersion in some models.
- Inaccuracies in the measurement and/or calculation of wind speed, solar and long-wave radiation, air temperature, cloud cover, and relative humidity.
- Effects of wind direction, fetch, channel width, sinuosity, bank and tree height.
- Effects of depth, turbulence, and lateral velocity distribution.
- Stability of the air moving over the stream.

#### 2.4.6 Convective Heat Flux, $Q_c$

Convective heat is transferred between air and water by conduction and transported away from (or toward) the air-water interface by convection

associated with the moving air mass. The convective heat flux is related to the evaporative heat flux,  $Q_e$ , through the Bowen ratio:

$$R = \frac{Q_c}{Q_e} = (6.19 \times 10^{-4}) p \left| \frac{T_s - T_a}{e_s - e_a} \right| \quad (2-68)$$

where  $R$  = Bowen Ratio

$p$  = atmospheric pressure, mb

$T_a$  = dry bulb air temperature,  $^{\circ}\text{C}$

$T_s$  = surface water temperature,  $^{\circ}\text{C}$

$e_s$  = saturation vapor pressure at the surface water temperature,  
mb

$e_a$  = vapor pressure of the overlying atmosphere, mb

The above formulation is used in the surface heat transfer budget of several models (e.g., U.S. Army Corps of Engineers, 1974; Brocard and Harleman, 1976).

#### 2.4.7 Equilibrium Temperature and Linearization

The preceding paragraphs present methods for estimating the magnitudes of the various components of heat transfer through the water surface. Several of these components are nonlinear functions of the surface water temperature,  $T_s$ . Thus, they are most appropriately used in transient water quality simulations where the need to predict temperature variations is on the time scale of minutes or hours. However, for long term water quality simulations or for steady state simulations, it is more economical to use a linearized approach to heat transfer. As developed by Edinger and Geyer (1965), and reported by Ryan and Harleman (1973), this approach involves two concepts, that of equilibrium temperature,  $T_E$ , and surface heat exchange,  $K$ , where  $H$  can now be written as:

$$H = K (T_s - T_E) \quad (2-69)$$

The equilibrium temperature,  $T_E$ , is defined as that water surface temperature which, for a given set of meteorological conditions, causes the surface heat flux  $H$ , to equal zero. The surface heat exchange coefficient,  $K$ , is defined to give the incremental change of net heat exchange induced by an incremental change of water surface temperature. It varies with the surface temperature and thus should be recalculated as the water temperature changes.

#### 2.4.7.1 Equilibrium temperature, $T_E$

The equilibrium temperature  $T_E$  is the temperature toward which every water body at the site will tend, and is useful because it is dependent solely upon meteorological variables at a given site. A water body at a surface temperature,  $T_w$ , less than  $T_E$ , will have a net heat input and thus will tend to increase its temperature. The opposite is true if  $T_w > T_E$ . Thus, the equilibrium temperature embodies all the external influences upon ambient temperatures.

Certain formulations for the equilibrium temperature have been developed which require an iterative or trial and error solution approach (Ryan and Harleman, 1973). An approximate formula for obtaining  $T_E$  has been developed by Brady et al. (1969) which has been shown to yield fairly accurate results:

$$T_E = \frac{Q_{sn}}{23 + f(W) (\beta + .255)} + T_d \quad (2-70)$$

where  $Q_{sn}$  = net short wave solar radiation, BTU/ft<sup>2</sup>/day

$T_d$  = dew point temperature of air, °F

$f(w)$  = empirical wind speed relationship

=  $17W_2$  (based on Lake Hefner data), BTU/ft<sup>2</sup>/day/mm Hg

$\beta$  = proportionality factor which is a function of temperature, mm Hg/°F

$W_2$  = wind speed at 2 meters above surface, mph

The expression for  $\beta$  is written as:

$$\beta = .255 - .0085 T^* + .000204 T^{*2} \quad (2-71)$$

where

$$T^* = \frac{1}{2} (T_w + T_d) \quad (2-72)$$

#### 2.4.7.2 Surface Heat Exchange Coefficient, K

The surface heat exchange coefficient, K, relates the net heat transfer rate to changes in water surface temperature. An expression for K developed by Brady et al. (1969), (and reported by Ryan and Harleman, 1973) is:

$$K = 23 + (\beta_w + .255) 17W_2 \quad (2-73)$$

where  $W_2$  = wind speed at 2 meters, mph

and  $\beta_w$  is evaluated at  $T_w$  based on Equation (2-62):

$$\beta_w = .255 - .0085 T_w + .000204 T_w^2 \quad (2-74)$$

Charts giving K as a function of water surface temperature and wind speed are given by Ryan and Stolzenbach (1972), assuming an average relative humidity of 75 percent. Shanahan (1984) presents a calculation procedure to determine  $T_E$  and K from average meteorological data.

#### 2.4.8 Heat Exchange with the Stream Bed

For most lakes, estuaries, and deep rivers, the thermal flux through the bottom is insignificant. However, as reported by Jobson (1980) and Jobson and Keefer (1979), the bed conduction term may be significant in determining the diurnal variation of temperatures in water bodies with depths of 10 ft (3m) or less. Jobson (1977) presents a procedure for accounting for bed conduction which does not require temperature measurements within the bed. Rather, the procedure estimates the heat



exchange based on the gross thermal properties of the bed, including the thermal diffusivity and heat storage capacity. The inclusion of this method improved dynamic temperature simulation on the San Diego Aqueduct and the Chattahoochee River.

#### 2.4.9 Summary

The previous section has presented a brief summary of the most frequently used formulations for surface heat exchange in numerical water quality models. These formulations are widely used and have been shown to work quite well within the normal range of meteorological and surface water conditions, provided a reasonably complete data base is available on meteorological conditions at the site of interest. Meteorological data requirements include atmospheric pressure, cloud cover, and at a known surface elevation: wind speed and direction, relative humidity, and wet and dry bulb air temperatures. Shanahan (1984) presents a useful summary of meteorological data requirements for surface heat exchange computations.

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